TOWARDS A MASS AND RADIUS DETERMINATION OF THE NEARBY ISOLATED NEUTRON STAR RX J185635-3754

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ABSTRACT

We discuss efforts to determine the mass, radius, and surface composition of the nearby compact object RX J185635-3754 from its multi-wavelength spectral energy distribution. We compute non-magnetized model atmospheres and emergent spectra for selected compositions and gravities, and discuss efforts to fit existing and new observational data from ROSAT, EUVE and the HST. The spectral energy distribution matches that expected from a heavy-element dominated atmosphere, but not from a uniform temperature blackbody. Non-magnetic light element atmospheres cannot be simultaneously reconciled with the optical and X-ray data. We extend previous studies, which were limited to one fixed neutron star mass and radius. For uniform temperature models dominated by heavy elements, the redshift z is constrained to be $0.3 \lesssim z \lesssim 0.4$ and the best-fit mass and radius are $M \approx 0.9 \text{ M}_{\odot}$ and $R \approx 6 \text{ km}$ (for a 61 pc distance). These values for M and R together are not permitted for any plausible equation of state, including that of a self-bound strange quark star. A simplified two-temperature model allows masses and radii up to about 50% larger, or a factor of 2 in the case of a black body. The observed luminosity is consistent with the thermal emission of an isolated neutron star no older than about 1 million years, the age inferred from available proper motion and parallax information.

Subject headings: dense matter — stars: neutron — stars: fundamental parameters — stars: individual (RX J185635-3754)

1. INTRODUCTION

It has long been hoped that stringent constraints could be placed on the equation of state (EOS) of dense matter from astrophysical measurements. Up to the present time, however, astrophysical constraints, most important of which is the establishment of a reliable minimum value for the neutron star's maximum mass (1.44 M_{\odot} from the binary pulsar PSR 1913+16 (Thorsett et al. 1994)), have not proven enormously useful in limiting the dense matter EOS. Although accurate masses of several additional neutron stars are available (Thorsett & Chakrabarty 1999), a precise measurement of the radius does not yet exist. There are observational estimates of radii, and limits to both mass and radius, such as those from Quasi-Periodic Oscillators (QPOs), X-ray bursts, and pulsars (see Lattimer & Prakash (2001) for a review). Even less information concerning the atmospheric composition is known. Spectra of neutron stars and pulsars have been fit with blackbody spectra (e.g., Paerels et al. 2000) and pure hydrogen atmospheres (e.g., Rutledge et al. 2001). The purpose of this paper is to present estimates of the mass, radius, and atmospheric composition of the nearby compact object, RX J185635-3754 based on model fitting to multiwavelength data. In spite of the lack of specific observed spectral features, we find that the star's redshift is constrained. The current generation of X-ray satellites will soon permit direct measurements of the gravitational redshift and surface composition.

Walter, Wolk & Neuhäuser (1996) discovered the X-ray source RX J185635-3754, and showed that its emergent flux was consistent with a blackbody (BB) spectrum with an effective temperature (observed at the Earth) of $T_{\infty} \simeq 57$ eV. The fortuitous location of the source in the foreground of the R CrA molecular cloud provides an upper limit to the distance D < 130 pc. The observed total flux implies that, for uniform surface temperatures, the so-called radiation radius $R_{\infty} = R/\sqrt{1-2GM/Rc^2}$ is only a few km. On this basis, they concluded it must be an isolated neutron star. Using the Hubble Space Telescope (HST), Walter & Matthews (1997) subsequently identified a source at optical (6060 Å) and near-ultraviolet (3000 Å) wavelengths, about 2.5 and 4 times brighter, respectively, than an extrapolation of a 57 eV BB into these bands. Neuhäuser, Thomas & Walter (1998) detected the object with V=25.7 using the 3.6 meter New Technology Telescope (NTT). Both the ultraviolet and red magnitudes have been confirmed by the subsequent HST measurements reported below. X-ray observations of RX J185635-3754, at energies greater than 1 keV, failed to show any evidence of a non-thermal tail or cyclotron emission lines. There is no

reported detection of a radio counterpart, and Walter et al. (1996) reported a limit to any X-ray variability of 7%. The optical flux rules out any active accretion disk or the presence of a binary companion with a stellar spectrum more luminous than about 10^{-7} L_{\odot} (in the optical band). We interpret these results to mean that there is neither significant accretion nor an intense magnetosphere. RX J185635-3754 appears to be a radio-quiet, isolated neutron star.

Three sets of observations with the HST Wide Field Planetary Camera 2 (WFPC2), taken within an interval of about three years, have yielded the proper motion (332 \pm 1 mas yr⁻¹) and parallax (16.5 \pm 2.3 mas), which corresponds, respectively, to a transverse velocity of about 108 ± 15 km s⁻¹ and $D=61^{+9}_{-8}$ pc (Walter 2001). This velocity and distance imply that the star originated $9\pm2\times10^5$ years ago in the Upper Scorpius OB association (Walter 2001). This interpretation is supported by a possible association with the runaway OB star ζ Oph, which also seems to have originated in the Upper Sco association at a similar time.

A space velocity greater than 10 km s⁻¹ virtually rules out the possibility that the neutron star has significant accretion (Madau & Blaes 1994). Unlike some other radio-quiet isolated neutron stars (Caraveo et al. 1996), which have pronounced evidence of non-thermal emission, this object may provide a clear view of the surface of a neutron star without complications from a magnetosphere or accretion. Ultimately, it holds the prospect that both its radius and mass, and the EOS of dense matter, might be tightly constrained.

The paper is organized as follows: The existing and new data available are presented in §2. In §3 we present a timing analysis and put limits on the modulation of the X-ray flux. In §4, we construct non-magnetic neutron star atmosphere models for selected chemical compositions and compare our results to previous studies. In §5, fits to the multiwavelength spectral energy distribution are computed, assuming that the neutron star's surface temperature is uniform. In §6, we explore the consequences of non-uniform thermal emission. Finally, in §7, we discuss some implications of our results, focusing on other aspects of the star and outlining other theoretical issues that might affect our interpretations.

2. SUMMARY OF OBSERVATIONS

2.1. X-ray Observations

2.1.1. ROSAT

ROSAT observed this target on three occasions (Table 1). The Position Sensitive Proportional Counter (PSPC) observation and the first of the two High Resolution Imager (HRI)

observations were discussed by Walter et al. (1996). The second HRI observation was obtained 3 years after the first for the purpose of studying the variability and proper motion of the target.

The HRI source counts were extracted from within a circle of radius 72 arcsec centered on the source centroid; the background was taken from a concentric annulus of inner and outer radii 72 and 216 arcsec, respectively. The net count rates were 0.555 ± 0.006 and 0.556 ± 0.005 counts s⁻¹ for the two observations; the source thus shows no evidence of long-term variability. Comparison of the position of this X-ray source with respect to 6 other X-ray sources in the HRI observations indicated that the target has a proper motion of 400 ± 200 milli-arcsec towards the south–east. This is consistent with the superior HST measurement (Walter 2001).

2.1.2. EUVE

The Extreme UltraViolet Explorer (*EUVE*; Haisch, Bowyer, & Malina 1993) pointed at the target for the better part of a month in June/July 1997 (Table 2), for a total of 582.6 ks. The target was clearly detected in the Deep Sky Survey (DSS) imager and in the Short Wavelength (SW) Spectrograph.

The DSS Image was taken in WSZ mode, which allows for pulse-height discrimination. Following the prescription of Christian (1995), we filtered the counts, selecting only those with pulse heights between 8500 and 15,500. This resulted in a 55% decrease in background counts with a loss of 14% of the source counts. The signal-to-noise ratio is commensurately increased by this pulse-height filtering.

We edited the list of 335 good time intervals, removing those intervals during which no counts were detected (there were 10 such intervals). At the beginning and end of each interval, we trimmed any time without photons provided the length of this period exceeded three times the expected time between counts, and also eliminated the frequent 2-3 minute period (extending up to 20 minutes during a few intervals) at the end of the observation without any counts. Doing so decreased the good exposure time by 14% to 499 ks.

The extracted pulse-height filtered data (mostly background) have a mean rate of about 1 count s⁻¹. We examined the instantaneous count rates (using a 20 second running mean) and eliminated all intervals where the total count rate was zero, or greater than 2 counts s⁻¹. The former represent times when the detector was not turned on; the latter are contaminated by scattered solar radiation from the limb of the Earth, or are contaminated by the particle radiation in the South Atlantic Anomaly. Following this filtering the net observing time is

478,696 s.

The corrected DSS count rate is 0.0429 ± 0.0003 counts s⁻¹. Within the uncertainties, this is consistent with the 3σ upper limit of <0.066 counts s⁻¹ reported by Walter et al. (1996), and with the detection at 0.028 counts s⁻¹ reported by Lampton et al. (1997). Both of those observations were made through the Lexan scanner filter, which is thicker and has lower throughput than the DSS imager. No other sources were detected in the DSS image.

To obtain the EUVE spectrum we created two-dimensional images in wavelength- θ space (θ is the angle off-axis perpendicular to the dispersion axis) from the pulse-height filtered data for each of the spectrometers. The dispersed spectrum was clearly present in the SW detector, with an excess of counts at $\theta=0$; as expected, there is no detection in the Medium or Long Wavelength detectors. We summed the counts in the SW image over angles $\theta=0\pm0.003^o$. We summed the background counts from $0.004^o-0.024^o$ above and below the spectrum, and smoothed them with a 3Å running boxcar prior to subtraction. We truncated the spectrum shortward of 76 Å. We divided the counts spectrum by the effective area of the SW detector (sw_ea.tab version dated 6 March 1997) and by the net exposure time, and corrected for the 14% signal loss in the pulse-height filtering. The net spectrum is shown in Figure 1. Uncertainties are computed from counting statistics. A continuum is clearly detected between about 75 and 95 Å, with net flux of $2.5\pm0.3\times10^{-13}$ erg cm⁻² s⁻¹ in this 20 Å interval.

2.1.3. ASCA

We observed the target with the Advanced Satellite for Cosmology and Astrophysics (ASCA) beginning on 1997 October 11 at 15:26 UT and ending at 13:34 UT on October 12 (sequence 25037000). The Solid-state Imaging Spectrometer (SIS) exposure time is 36.6 ks; the Gas Imaging Spectrometer (GIS) exposure time is 41.4 ks. We observed with a single CCD in bright mode.

The target was detected in the two SIS detectors, with 480±33 counts. There is a marginal detection in the lowest channels of the GIS. In neither case is there any detection at energies above 1.5 keV. We extracted the spectra using both FTOOLS and software written in IDL. The background is the mean of background regions extracted from around the source. The net SIS spectrum is shown in Figure 2.

We fit the SIS spectrum to a BB using Version 10 of XSPEC. The best-fit BB parameters $(T_{\infty} = 60\pm_{25}^{20} \text{ eV}, n_{H,20} < 70)$ are consistent with, though less well-determined than, the ROSAT PSPC fit (Walter et al. 1996). The total observed flux is $2.2 \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$.

The low count rate did not warrant more detailed spectral fitting. We show the net GIS spectrum in Figure 2. Given the low S/N of the detection and the uncertain response of the GIS below 1 keV, we did not attempt to fit these data, other than to note that the data are consistent with a soft BB spectrum. Consequently, we do not use these data in the multispectral fits.

There is no evidence for any line emission at energies above 1.5 keV. Such line emission might be expected from a magnetized neutron star accreting from the interstellar medium (Nelson et al. 1995). Neither is there any evidence for a hard tail, as would be expected in an accreting source due to comptonization of the infalling electrons (Zampieri et al. 1995). The 2σ upper limits on the 1.5–10 keV flux are 1.01×10^{-5} photons cm⁻² s⁻¹ in the SIS and 3.8×10^{-5} photons cm⁻² s⁻¹ in the GIS.

No other X-ray sources were detected in the SIS. The source RX J185533-3805 was detected in the GIS (it is outside the SIS field of view). Its spectrum can be fit either with a 700 eV BB or an α =1.9 power law. The nature of this object is not known; there is no obvious optical counterpart.

2.2. UV/Optical Photometry

2.2.1. Broadband Imaging from the HST

We observed and detected the target with the WFPC2 using the F606W, F450W, F303W, and F178W filters (Table 3). The three F606W images have been used to determine the parallax and proper motion of the target (Walter 2001). Here were discuss the broadband photometry.

The F606W and F450W images were dithered using the standard 0.5" diagonal pattern. We coaligned and median-filtered the images to remove cosmic rays and, in the dithered images, the warm pixels. The target was clearly visible in all images.

We determined the net counts using aperture photometry. The background is determined in an annulus of selectable width surrounding the target, with the inner edge of the background annulus offset from the extraction radius by a selectable amount. The mean background level is determined iteratively by rejecting all pixels more than 3σ from the median. The net extracted counts are highly sensitive to small fluctuations in the mean background, which depends on the location of the background annulus. In the F606W filter, the substantial background causes the formal S/N of the extracted counts to fall below unity for extraction radii of 11-12 pixels. The highest significance of the measured flux occurs for a

2 pixel (0.09 arcsec) extraction radius, which necessitates a substantial aperture correction. We determined the aperture correction empirically using nearby bright stars. We actually measured the source flux within a 3 pixel (0.14 arcsec) because the aperture correction is more subject to errors arising from the exact location of the star relative to the central pixel when using the 2 pixel aperture. We converted the count rates to fluxes using the conversion factor given in the PHOTFLAM keyword. We applied the empirical aperture correction to the standard 11 pixel radius, and then added the 0.1 magnitude correction to an infinite aperture. All geometric corrections are negligible. We corrected for the charge-transfer efficiency following (Dolphin 2000). We apply decontamination corrections (1–3%) to the F300W and F170W magnitudes. We list these measured fluxes in Table 3, along with the formal significance (in σ) for the detection. The uncertainty in the F606W and F450W fluxes is dominated by the uncertainty in the empirical aperture correction.

2.2.2. Other Optical Observations

We observed the target with the NTT at the European Southern Observatory (ESO) on the night of 1997 August 9/10. The night was photometric, but the seeing conditions (varying between 1.1 and 1.4 arcsec) required us to use the ESO Multi-Mode Instrument (EMMI) instead of the SUperb Seeing Imager (SUSI). We used the EMMI red CCD # 36, with the V-band filter ESO # 606 and with the R-band filter ESO # 608. Throughout the night we observed Landolt standard star fields for photometric calibration.

We took several images to reduce the risk of cosmic ray contamination and placed the expected target position onto slightly different areas on the chip in each exposure to avoid problems with bad pixels. After bias and flat field correction, we added the images using standard MIDAS procedures to construct the final V-band image with a total exposure time of 150 minutes. This image is shown in Neuhäuser, Thomas & Walter (1998). The target is clearly detected with a S/N of 18 inside the ROSAT error circle at a position consistent with the HST position.

We measured the V magnitude to be 25.70 ± 0.22 (using the MIDAS command magnitude/circle), and $V \simeq 25.72$ (using the MIDAS Romafot package). This is consistent with the HST F606W flux.

We also obtained images in the Kron-Cousins R-band, totaling two hours of exposure, towards the end of the night with air masses between 1.20 and 1.28, and seeing between 1.5 and 1.9 arcsec. We did not detect the target, with an upper limit of $R \simeq 24.5$ mag.

2.2.3. UV Spectrophotometry

We obtained a far UV (1150-1720 Å) low dispersion (G140L) spectrum using the *HST* Space Telescope Imaging Spectrograph (STIS; Woodgate et al. 1998) on 23 and 26 October 2000. We placed the target in the F52X0.5 slit using a blind offset from star J (Walter et al. 1996). The total integration time is 26900 seconds (10 spacecraft orbits). The position angle of the slit was 359.4° and 358.5° on the two days.

We coadded the individual 2-dimensional spectra to produce a net spectrum. There was no evidence for any spatial extent in this coadded image. We extracted the source flux over 7 spatial pixels (± 0.085 arcsec). We extracted the background over 10 spatial pixels (0.244 arcsec), both above and below the source, with an offset of 2 spatial pixels between the extraction regions. We subtracted the background spectrum, and then smoothed the net spectrum. The spectral shape is consistent with a blackbody. There are no statistically significant spectral features. The largest spectral feature, at 1595Å, has an equivalent width of about 7 ± 3 Å. We discuss this spectrum further in §6.

3. Timing

Walter, Wolk & Neuhäuser (1996) reported the failure to detect a rotation period in the ROSAT PSPC observation of RX J185635-3754. The detection of a period would be significant, in that it would require that the surface flux distribution be non-uniform, or, if the surface is axi-symmetric about some axis, that the axis be inclined with respect to the rotation axis. In addition, the period would let one make inferences about the rotational history of the system, and search for a secular spindown. On the other hand, the failure to detect rotational modulation could place constraints on the magnitude of surface inhomogeneities. We report here on a search for the pulsation period in the three ROSAT data sets.

We searched each of the three *ROSAT* observations (Table 1) for evidence of periodic modulations. We used the FTOOLS tasks *abc* and *bct* to generate and apply the barycentric corrections to the data event lists. We then searched the data both by FFT and period-folding techniques. We verified the techniques by recovering the correct periods and modulation profiles for the Crab pulsar and the 8.4 s period X-ray pulsator RX J0720-31 (Haberl et al. 1997).

The *ROSAT* data are not continuous, but consist of a number of short segments. We edited out the first 30 s of each segment, to mitigate against the occasional effects of delayed high voltage turn-on, and ignored any data segment shorter than 300 s.

3.1. Power-Spectrum Analysis

We generated an FFT of the data in each continuous time interval and then added the individual power spectra to generate a net power spectrum for the complete data set. In this way we did not have to deal with the data gaps. By ignoring the shorter intervals, we reduce the total exposure time by about 10%, but with little loss in sensitivity. This reduced the net exposure time for RORs 400612 and 400864 by 15% and 7%, respectively; the PSPC observation was not affected.

We searched for periods between 0.1 and 200 s. No obvious signal stands out in the power spectra of the three ROSAT observations. (There is strong power in the PSPC power spectrum at 80 seconds, but this is most likely attributable to the spacecraft wobble, as the source is near the inner supporting ring. No such power is seen in the HRI data.) If there is a true rotational signal present, it will be present in all three data sets, but there are no coincidences amongst the 20 strongest peaks in the three data sets. To determine the significance of the strongest peaks in the power spectra, we generated 1000 (HRI) and 4000 (PSPC) simulated data sets with the mean count rate of the target. The photon arrival times were uniformly distributed within the good observation times. We added Poisson noise to the photon arrival times, and generated power spectra. The strongest peak in the PSPC power spectrum has a 69% likelihood of arising by chance. Similarly, the strongest peak in the HRI power spectrum has a 90% likelihood of arising by chance.

3.2. Period-Folding Analysis

In a second search for a rotational period, we folded the data on trial periods ranging from 0.1 to 12 s. The sampling periods were chosen such that no period would be smeared by more than 10% over the up-to-6-day length of the observations. As we are searching for a smooth (sinusoidal) modulation, and not a sharp pulse, this is acceptable. The data were folded into 20 phase bins at each trial, and then tested against the null hypothesis using a χ^2 test. For each of the three data sets, only a single period was found to yield a probability less than that at which one would expect a single false alarm, but as the three periods do not agree, we conclude that there is no detected period.

3.3. Limit to the Modulation Amplitude

In order to determine our sensitivity to low amplitude periodicities, we generated simulated data sets with a sinusoidal modulation of known amplitude. We varied the period of

the modulation between 0.1 and 20 s, and the amplitude from 4 to 15%. We applied the power-series and period-folding analyses to each simulated data set. We considered the simulation a success if the input period was recovered to within the uncertainty of the measured period, and a failure otherwise.

The observations recover the correct period over 50% of the time for amplitudes greater than about 6% at periods shorter than 5 s, and recover periods with amplitudes greater than 5% for periods between 5 and 20 s. Thus we place an upper limit of 6% on the amplitude of any rotational modulation in the soft X-rays. This is about half the amplitude of the pulsation seen in RX J0720-31 (Haberl et al. 1997).

4. ATMOSPHERIC MODELING

Neutron star atmosphere models for low magnetic fields (< 10¹¹ G) were first developed by Romani (1987), and followed later by others (Miller 1992; Rajagopal & Romani 1996; Zavlin, Pavlov & Shibanov 1996). The models cover different compositions, such as pure hydrogen, helium, carbon, nitrogen and iron, as well as a solar mixture. The strong 10^{12–13}G magnetic fields of pulsars suggests that most, if not all, neutron stars should have similarly strong fields. However, detailed atmosphere models with strong magnetic fields are only available for hydrogen (Zavlin et al. 1995), mainly because reliable opacities and EOS have not yet been developed for heavier elements. For heavy-element dominated atmospheres, only approximate treatments of magnetic Fe atmospheres exist (Rajagopal, Romani & Miller 1997), and the results show that the spectra are globally much closer to a blackbody than for light element atmospheres. We focus on non-magnetized models in this paper, which affords a comparison with earlier work, and also provides a benchmark for future calculations with magnetized atmospheres.

We selected a small set of representative chemical compositions: pure H, pure He, pure Fe, and a mixture of heavy elements labelled Si-ash. Table 4 identifies the specific composition of the Si-ash case, which was chosen to mimic the composition at the end of silicon burning (Arnett 1996) and which might be typical of the matter initially accreted onto a newly formed neutron star.

We note that only a tiny amount of accreted matter from the interstellar medium, $\sim 3 \times 10^{-4} \text{ g cm}^{-2}$ of H, corresponding to an accretion rate of $\dot{M} = 10^{-30} \text{ M}_{\odot} \text{ yr}^{-1}$ for a million years, is needed to render the atmosphere optically thick to H at an energy of 0.25 keV. Gravitational settling ensures that heavy elements settle out of the atmosphere and will not contribute to the emergent flux in this event. Nevertheless, it is not certain that all

neutron stars can accrete sufficient amounts of H for this to occur. For example, magnetized, rotating neutron stars will be in the ejector or propeller phase (Colpi et al. 1998), and are not expected to accrete. Therefore, it is a distinct possibility that some neutron star atmospheres are dominated by heavy elements.

4.1. Model Computation

The atmosphere models have been calculated using standard techniques for the construction of radiative, local thermodynamic equilibrium, plane-parallel atmospheres (Romani 1987; Miller 1992; Rajagopal & Romani 1996). The opacities and EOS were obtained from the Los Alamos opacity project (http://www.t4.lanl.gov), prepared in tabular form. A reference optical depth (chosen to be a Rosseland mean) grid of 120 points is chosen for the range from 10⁻⁸ to 10², and photon frequencies are gridded logarithmically in 190 levels from approximately 1 eV to 10 keV. Initially, hydrostatic equilibrium is imposed for a temperature profile derived from the gray opacity approximation. The flux is then computed at each level of the atmosphere, and a variation of the Lucy-Unsöld procedure is used to calculate the temperature corrections needed to satisfy flux constancy. This variant utilizes the flux and Eddington factors evaluated by numerical integration of the current profiles, instead of using constant values of 1/2 and 1/3 as in the original method. This modification speeds convergence and accuracy. With the temperature corrections, hydrostatic equilibrium is reestablished, new fluxes are evaluated, and the procedure is repeated until flux constancy is reached to within 0.5% throughout the atmosphere.

The monochromatic opacity used in the calculations is the sum of the absorption (α_{ν}) and scattering (σ_{ν}) opacity. The frequency dependent source function can be written as

$$S_{\nu} = \frac{\sigma_{\nu} J_{\nu} + \alpha_{\nu} B_{\nu}}{\alpha_{\nu} + \sigma_{\nu}} = (1 - f_s) B_{\nu} + f_s J_{\nu}$$
 (1)

where J_{ν} is the mean intensity, B_{ν} the Planck function at the local temperature, and $f_S = \sigma_{\nu}/(\alpha_{\nu} + \sigma_{\nu})$ is the ratio of the scattering opacity to the total opacity. This standard form of the source function has been used by other authors (e.g. Zavlin et al. 1996), and implies the isotropic scattering approximation (no angle dependence in the opacities) which has been shown to be accurate even in the case of scattering dominated atmospheres (e.g. Mihalas 1978). Notice, however, that at the temperatures of interest to our work the true absorption opacities are several orders of magnitude larger than the scattering opacity $(f_s \ll 1)$, especially for heavy element atmospheres. Thus the use of an even simpler source $S_{\nu} = B_{\nu}$, as in Rajagopal & Romani (1996), will suffice to obtain accurate spectra. This situation can be different for low temperature (10⁵K), light element (H) atmospheres, where

scattering processes contribute significantly and corrections from anisotropic scattering might be significant.

Note that the redshift, which is significant in neutron star atmospheres, does not enter into the calculation of the emergent flux. The flux observed at the Earth, however, must be corrected for the surface redshift of a neutron star.

4.2. Comparison with Previous Work

In Figure 3 we show the spectral fluxes of the emergent radiation for selected effective temperatures and chemical compositions. The BB fluxes are also shown for comparison. In the left panel we compare the BB flux (dotted lines) with that of pure Fe atmospheres (solid lines), for the indicated values of $\log T_{eff}$. The range chosen, 5.25 $< \log T_{eff} < 6.25$, allows a comparison with Figure 2 in Rajagopal & Romani (1996). The agreement between our results with those of Rajagopal & Romani (1996) is good, despite the fact that Rajagopal & Romani (1996) used opacities and EOSs from the OPAL project (Iglesias & Rogers 1996; Rogers, Swenson & Iglesias 1996). In the right panel, we compare BB (dotted lines) and pure Fe (solid lines) fluxes with those for pure H (dash-dotted lines) and Helium (dashed lines). For the right panel, we chose specific values of T_{eff} to allow a comparison with Figure 5 in Zavlin, Pavlov & Shibanov (1996). Although the agreement between H and He models in both works is excellent, there are deviations apparent in the high energy tail of the Fe models, especially for high temperatures. These differences do not appear to be explained by our use of Los Alamos opacities, since both Zavlin et al. (1996) and Rajagopal & Romani (1996) employed OPAL opacities. Pavlov (private communication) recently informed us that Zavlin et al. (1996) contained an error in the implementation of Fe opacity tables that might explain the observed differences, and that corrected Fe atmosphere models showed excellent agreement with those of Rajagopal & Romani (1996), despite the use of different algorithms for the atmospheres.

4.3. The Effect of Gravity

Aside from the effective temperature and the composition, the local gravitational acceleration

$$g = \frac{GM}{R^2 \sqrt{1 - 2GM/Rc^2}} \tag{2}$$

potentially affects the emergent spectra. This parameter is nearly constant throughout the atmosphere because both its mass and thickness are negligible compared to the star's total mass and radius. That the thickness of the atmosphere is negligible compared to the stellar radius also justifies treating the atmosphere as being plane-parallel, and this approximation is generally even more accurate for neutron stars than for normal stars.

Figure 4 shows the variations with g of the emergent (unredshifted) spectra of an Fe atmosphere with $\log T_{eff} = 5.6$. The solid line is the emergent spectra of a canonical neutron star ($M = 1.4 \text{ M}_{\odot}$ and R = 10 km, which corresponds to $g_{14} = 2.43$, where g_{14} is the gravitational acceleration in units of $10^{14} \text{ cm s}^{-2}$). The emergent spectra from two other configurations, one more compact (R = 8 km, $g_{14} = 4.2$) and the other less compact (R = 12 km, $g_{14} = 1.6$), are also shown. For reference, the BB spectrum, which does not depend upon g, is also shown. The effects of gravity on the emergent spectra, in this energy range, are obviously small. The chief differences are in the vicinity of high-energy spectral lines and in the high-energy tail, but these are too small, given the limited resolution of the existent X-ray spectra, to affect parameter constraints.

For the range of g above, the corresponding gravitational redshifts z are 0.235 < z < 0.348, where z is defined through

$$(1+z)^{-1} = \sqrt{1 - \frac{2GM}{Rc^2}}. (3)$$

The redshift produces measurable changes in the observed fluxes, as shown below. Thus, although separate constraints on g and z are possible in principle, only z is meaningfully constrained by atmospheric modelling at this time. In the following sections detailing the results of spectral fitting, the gravity is not treated as a free parameter.

5. FITTING TO UNIFORM TEMPERATURE MODELS

In this section, we focus on the estimation of parameters from spectral fitting, assuming that the effective temperature is uniform across the stellar surface. However, it should be noted that a neutron star with a moderate and non-uniform surface magnetic field, for example, possibly has a non-uniform surface temperature as well, owing to the changing conductivity of the surface layers. In addition, hot spots on the surface due to accretion or other phenonema may exist. The consequences of allowing the temperature to vary across the surface of the star is explored in §6.

The relevant parameters for spectral fitting include the atmospheric composition, the temperature observed at the Earth $T_{\infty} = T_{eff}/(1+z)$, the redshift z, the interstellar medium

column density n_H , and the angular diameter R_{∞}/D . The normalization factor $(R_{\infty}/D)^2$ is the solid angle subtended by the star's surface visible at a distance D. The definition of $R_{\infty} = R/\sqrt{1 - 2GM/Rc^2}$ arises from the blackbody relations, $F = 4\pi (R/D)^2 T_{eff}^4$ and $F_{\infty} = 4\pi (R_{\infty}/D)^2 T_{\infty}^4$, where F_{∞} is the flux observed at the Earth. Note that the actual radius of the star R must be less than R_{∞} , and that a measured value of R_{∞} also implies an upper limit to the mass, $M < (c^2/G)R_{\infty}/\sqrt{27} \approx 0.13 M_{\odot}(R_{\infty}/km)$.

The presence of an atmosphere can alter the thermal emission from a neutron star substantially from a pure Planck spectrum. For a pure Planck spectrum, the redshift contributes only to an overall scale factor, so that it no longer serves as a parameter and no information concerning it can be obtained from fitting. For a realistic atmosphere, however, the presence of spectral features, such as the high energy cut-off observed in heavy-element dominated atmospheres in §4, makes the redshift a measurable parameter. In addition, although we do not consider them in any detail in this paper, strong magnetic fields, if present, could have an appreciable effect on the results of atmospheric fitting (Zavlin et al. 1995, Rajagopal et al. 1997).

5.1. Fitting the ROSAT PSPC Data

We first examine the X-ray data from *ROSAT* since this is where the bulk of the flux is found. Walter et al. (1996) discuss a blackbody fit to the PHA spectrum. We have reextracted the spectrum using FTOOLS and fit the data using XSPEC V11.0. We tabulated the neutron star atmospheric model spectra as external additive tables for input into XSPEC, according to the standard OGIP directives. We fit only PHA channels 11-100 (0.11 to 1.0 keV). There is negligible flux at higher energies, and the calibration in the lowest channels is questionable.

5.1.1. Fits at the Nominal Redshift

To facilitate comparison with previous work, we first fixed the mass $M=1.4~\rm M_{\odot}$ and radius $R=10~\rm km$, which gives $z=0.305~\rm and~R_{\infty}=13.05~\rm km$. The angular diameter R_{∞}/D , computed from the normalization of the model fits, implies a value for D, which can be compared to the recently measured parallactic value 61^{+9}_{-8} pc (Walter 2001). Discrepancies between this and the predicted values indicate that the atmosphere model, or the assumed redshift, are incorrect. Our results for the BB, H, and He atmospheres substantially agree with those previously published (Walter et al. 1996, Pavlov et al. 1996). Our Fe atmosphere

fit also gives a considerably lower temperature and higher column density than that of Pavlov et al. (1996). The difference is probably explained by deviations in the hard X-ray tails of the emergent spectra noted in the previous section. In Table 5 we summarize the optimum fits to the ROSAT PSPC data for various assumed compositions.

We also computed models with the Si-ash composition (Table 4), the results of which are intermediate between those of Fe and BB models, probably because the presence of many elements and more absorption lines causes a larger energy redistribution than in a pure Fe atmosphere. The results from Table 5 indicate that the Si-ash and Fe spectra are reasonably consistent with the measured distance.

Note that none of the fits are formally acceptable. The best fit, that of the blackbody spectrum, is rejected at the 99.5% confidence level. The best fits for the various models are not statistically distinguishable: no single one is preferable. Figure 5 shows the best fit Fe atmosphere (which is formally the worst of our fits). The residuals below 0.2 keV are present in all the fits, and may be attributable to uncertainties in the PSPC response at low energies (the PSPC response is not measured shortward of the Boron $K\alpha$ edge at 0.188 keV, but has been extrapolated based on the known characteristics of the detector). The high point at 0.3 keV is a statistical fluctuation. The differences between the fits lie primarily between 0.7 and 0.9 keV, where the Fe and Si-ash models underpredict the flux.

The predicted Si-ash and Fe spectra include a number of spectral lines. These lines are not directly visible with ROSAT, owing to the poor energy resolution ($\Delta E/E \approx 60\%$) of the PSPC detector; however, they alter the spectral shape sufficiently that they may constrain the gravitational redshift. These absorption lines should be detectable in CHANDRA and XMM-Newton grating spectra.

5.1.2. The Effect of Varying the Redshift

The gravitational redshift z is a function of M/R, and so to fix z = 0.305 is to presuppose this ratio (0.139 M_{\odot}/km). While the BB results are unaffected by variations in redshift, the strong absorption feature near 0.6 keV in the heavy element spectra, which acts as a high energy cutoff to the emergent spectra, can in principle be constrained by the X-ray spectra.

We used XSPEC to fit a series of redshifted models to the PSPC data. We found that the best fit to the data using the pure hydrogen model is not significantly affected by the gravitational redshift, probably because there are no discrete features in the emergent spectrum. For both the Fe and Si-ash models we could reduce the value of χ^2 to that seen in the BB and H spectra by varying z, but the improvements are not significant ($\Delta \chi^2 \approx 20$ –30

for 86 degrees of freedom). The best fits with z as a free parameter are presented in Table 5.

For both the Fe and Si-ash models, we find a strong correlation between the best fit values of n_H and T_{∞} (Figure 6). The general trend is that n_H increases, and T_{∞} decreases, with decreasing z For a given z, the Si-ash models give systematically larger temperatures (by about 6.5 eV) than the Fe models.

Quantitatively, the constraint between n_H and z appears to be relatively insensitive to composition, and can be expressed as

$$n_{H,20} = (4.9 \pm 0.3)(1+z)^{-1} - (1.9 \pm 0.2).$$
 (4)

Separate correlations between T_{∞} and n_H can be extracted as well (Figure 7). The redshift z decreases from right to left in this figure. The result for the BB model is also shown (diamond) for reference. The solid lines represent the quadratic fits

$$T_{\infty} = \left\{ \begin{array}{ll} (157 \pm 2) - (96 \pm 3)n_{H,20} + (17 \pm 1)n_{H,20}^{2} \text{ eV} & \text{Fe} \\ (162 \pm 4) - (91 \pm 4)n_{H,20} + (15 \pm 1)n_{H,20}^{2} \text{ eV} & \text{Si - ash} \end{array} \right\}.$$
 (5)

For a given redshift the 95% confidence contours are ellipsoids with a typical vertical extent of about 5 eV above and below these curves.

The constraints in equations (4) and (5) arise from the competition between T_{∞} and n_H in fitting the low-energy side, and between T_{∞} and z in fitting the high-energy side, of the X-ray spectrum. A further constraint arises from the fact that the spectrum can be approximated by a blackbody, so the total flux, most of which is emitted in soft X-rays, is proportional to $T_{\infty}^4(R_{\infty}/D)^2$. We parameterize the effect of interstellar extinction very simply as $\exp(-a n_H)$, where a is a constant, so that the observed flux is proportional to $T_{\infty}^4(R_{\infty}/D)^2 \exp(-a n_H)$. This works because the strong interstellar extinction in the extreme ultraviolet coincides with the peak of the thermal spectrum. By so doing, we find the following effective constraints for each composition:

$$T_{\infty}^{4}(R_{\infty}/D)^{2} = \begin{cases} (6147 \pm 839) \exp[(1.46 \pm 0.07)n_{H,20}] \text{ (eV)}^{4} & \text{Fe} \\ (3719 \pm 842) \exp[(1.58 \pm 0.10)n_{H,20}] \text{ (eV)}^{4} & \text{Si - ash} \end{cases}$$
(6)

where R_{∞} is in km and D is in pc.

The fits to the PSPC data provide interesting contraints on the parameters z, n_H , T_{∞} , and R_{∞}/D . However, the PSPC spectra alone can be used neither to decide the atmospheric composition, nor to exclude any realistic values for the gravitational redshift. For that we require the additional leverage provided by the longer wavelength data.

5.2. Fits to Multiwavelength Observations

Figure 3 shows that the spectral energy distributions are quite sensitive to both the composition and the temperature. In particular, heavy element atmospheres have more opacity at short wavelengths than do light element atmospheres, deviate less from a blackbody, and have a smaller optical-to-X-ray flux for a given T_{eff} . The X-ray band is sensitive to the exponential end of the Planck distribution, as modified by interstellar extinction. Primarily because of the low spectral resolution of the PSPC, but also because the low temperature ensures that the peak of the spectral energy distribution is highly absorbed, the extant X-ray data are not useful for discriminating between the various models. However, extrapolation of the optimum X-ray fits into the optical region provides much greater leverage. In this regime all the models can be approximated as the lightly reddened Rayleigh-Jeans tail of a blackbody, with the observed intensity constraining $T_{\infty}(R_{\infty}/D)^2$.

Here we incorporate into our analysis the EUVE, NTT, and HST data summarized in §2. The dashed lines in Figures 8–11 show the extrapolations of the best z=0.305 (Table 5) ROSAT PSPC fits to the full spectral energy distribution. The ROSAT data themselves are not shown because the response matrix redistributes the energy and a model independent flux cannot be recovered. For consistency with the XSPEC calculations, we use the Morrison & McCammon (1983) cross sections for the EUVE extinction. The EUVE data are generally consistent with the ROSAT observations; thus they are extremely useful in the energy range 0.1–0.18 keV in which the ROSAT PSPC is not well calibrated.

These figures show that for the best fits to the X-ray data, the BB model underpredicts the optical flux by a factor of 2.2 ± 0.4 , the Fe model overpredicts the optical flux by a factor of $4^{+.8}_{-.6}$, the Si-ash model prediction is just about right (overprediction by $1.3^{+.3}_{-.2}$), and the H and He (not shown) models overpredict the optical flux by a factor of 30. This suggests that a single-temperature Si-ash atmosphere may be appropriate. Simultaneous fitting of the long and short wavelength data will yield better constraints on the phyical parameters.

5.2.1. Fits at the Nominal Redshift

We fit the multiwavelength data in the following manner: We used XSPEC to generate a grid of fits in the R_{∞}/D , T_{∞} plane at the nominal redshift, z=0.305. At each point we fit the absorption column and generate a goodness of fit (χ^2) statistic for the PSPC data. At each point in the grid we then generate the appropriate model atmosphere and determine the goodness of fit statistic for the optical and UV data. We weight each of the 4 data sets (PSPC, EUVE, HST/STIS, and UV/optical photometry) equally. At each point we

sum the reduced χ^2 for each of the 4 data sets to determine a best fit region. These χ^2 grids are shown in Figures 12–15. The *EUVE* and PSPC data define a region that is most sensitive to the absorption column. The *EUVE* data are consistent with the PSPC data, but do not further constrain the fit. The optical photometry and the STIS spectrum, lying on the Rayleigh-Jeans tail of the spectral energy distribution, define a region of approximately constant R_{∞}/D . The best fits generally occur near the intersections of these two regions. The resulting constraints are given in Table 6. These best overall fits are shown as the solid lines in Figures 8–11. We quantify the goodness of the joint parameters by multiplying the likelihoods of the most contraining data, the PSPC and optical data. These are given in Table 6. We determine the luminosities (Table 6) by integrating the overall best fit models, assuming a 61 pc distance.

Because the optical and UV fluxes define the Rayleigh-Jeans tail of a hot blackbody, we observe the quantity $T_{\infty}(R_{\infty}/D)^2\phi$, where ϕ is the transmission though the interstellar medium. We use the mean flux in the STIS spectrum and the mean flux in each of the 4 WFPC2 filters. Each data point independently fixes a value for $T_{\infty}(R_{\infty}/D)^2\phi$ on the assumption that the observed flux is proportional to λ^4 . Minimizing the scatter in $T_{\infty}(R_{\infty}/D)^2\phi$ with respect to n_H (using the Seaton (1979) extinction curve) yields $n_H=1.6\pm0.2\times10^{20}$ cm⁻² and $T_{\infty}(R_{\infty}/D)^2=0.60\pm0.01$. We also fit the data to determine the following constraints:

$$T_{\infty}(R_{\infty}/D)^{2} = \begin{cases} 0.59_{-.01}^{+.03} & \text{BB} \\ 0.80_{-.05}^{+.09} & \text{Fe} \\ 0.74 \pm 0.04 & \text{Si - ash} \end{cases}$$

where T_{∞} is in eV, R_{∞} is in km and D is in pc. The BB result agrees with the analytic result. That the Fe and Si-ash results are larger than the blackbody result is a consequence of different best-fit values of n_H (greater extinction requires a larger $T_{\infty}(R_{\infty}/D)^2$ to produce the same observed flux), with perhaps some contribution from small deviations from a pure Rayleigh-Jeans tail. These constraints, together with those in equations (4–6), in principle allow a unique determination of the four fitting parameters for each model.

A non-magnetic Si-ash model provides an acceptable fit in all wavelength bands. This model gives $R_{\infty}/D=0.13$ km pc⁻¹, or $R_{\infty}=7.8\pm1.2$ km for the 61 pc distance. The likelihood that the optical and X-ray parameters are identical is 53%.

The best multiwavelength BB model yields a poor fit to the PSPC data. Figure 1 shows that the best multiwavelength fit provides a better fit to the EUVE data (both fits are acceptable at the 1σ level). This may be attributable to uncertainties in the ROSAT PSPC calibration at low energies. Note that the EUVE detection corresponds to PSPC channels 11-16, which seem to show systematic deviations in Figure 5. The best fit to the X-rays alone underpredicts the optical flux, but this can be made up with emission from a component too

cool to contribute significantly to the X-ray emission. The likelihood that the optical and X-ray parameters are identical is 0.03%. A multi-temperature blackbody model can yield an acceptable fit (see §6).

The best non-magnetic Fe model provides an unacceptable fit in the X-ray band, but its parameters are similar to the Si-ash case. Unlike the case of the BB, the best X-ray fit overpredicts the optical flux. An overprediction cannot be lessened by adding additional cooler regions. Additional opacity, as in the Si-ash atmosphere, brings the star closer to the blackbody limit and would reduce the optical overprediction. As we discuss in $\S 5.2.2$, the overpredicted optical flux can be lessened by an increase in z.

The uniform temperature non-magnetic hydrogen model can be excluded by these data (Figures 11 and 15).

5.2.2. The Effect of Varying the Redshift

The BB models are not sensitive to the redshift. The hydrogen model is sensitive only in that the observed wavelength of H I Lyman α depends on z. The model predicts that the line is in absorption with an equivalent width of about 25Å. We see no significant absorption lines in the far-UV, with a limiting equivalent width of about 10Å, excluding H-dominated model atmospheres in the range 0.07 < z < 0.38. The He atmosphere similarly exhibits no significant redshift dependence.

The heavy element atmospheres are affected by the redshift. We discussed the effects of the redshift on the X-ray spectral fits earlier. At long wavelengths the redshift dependence appears because the flux is proportional to $T_{\infty}(R_{\infty}/D)^2$ (with a small correction for reddening), and T_{∞} is a function of z. Higher redshifts, and larger values for T_{∞} , require smaller values for R_{∞}/D . In both the X-ray and optical regimes we obtain relations between R_{∞}/D and z, as shown in Figure 16. The intersections of these regions constrain z and R_{∞}/D . For the Si-ash model, we find that $0.124 < R_{\infty}/D < 0.143$ and 0.305 < z < 0.344. For a pure Fe atmosphere the constraints are $0.124 < R_{\infty}/D < 0.149$ and 0.346 < z < 0.405. Note that the values of R_{∞}/D are nearly identical. These best fits for the Si-ash and Fe atmospheres, summarized in Table 7, are shown as the dotted lines in Figures 9 and 10.

The uniform temperature non-magnetic hydrogen model cannot be saved by varying z because T_{∞} and R_{∞} do not have any significant redshift dependence.

5.3. The Best Fit Parameters

Having parameterized the constraints from the X-ray fits (4–6), together with the optical constraint in equation (7), in principle we can uniquely determine the four fitting parameters. This provides a consistency check on the parameters derived from the multiwavelength χ^2 minimization (Table 6).

The best parameters for the heavy element atmospheres are summarized as:

$$\begin{cases}
R_{\infty} = 8.2 \pm 1.3 \text{ km} & R = 6.0 \pm 1.5 \text{ km} & M = 0.95^{+0.06}_{-0.16} M_{\odot} \\
z = 0.39 \pm 0.12 & n_{H,20} = 1.7 \pm 0.1 & T_{\infty} = 44 \pm 4 \text{ eV}
\end{cases}$$
 Fe
$$\begin{cases}
R_{\infty} = 7.8 \pm 1.3 \text{ km} & R = 6.0 \pm 1.4 \text{ km} & M = 0.84 \pm 0.07 M_{\odot} \\
z = 0.31 \pm 0.12 & n_{H,20} = 1.8 \pm 0.2 & T_{\infty} = 45 \pm 6 \text{ eV}
\end{cases}$$
 Si – ash

The quantities R_{∞} , R, and M all scale linearly with D.

We compare these analytic values to the fits to the multiwavelength data in Table 7 for the two heavy element atmosphere models. The uncertainties on the analytic fits exceed those determined from the data directly, primarily because the uncertainty in the direct measurement of the temperature is significantly less than the scatter in the quadratic fit to T_{∞} as a function of n_H . Within their uncertainties the derived values of the parameters are in excellent agreement.

5.4. Comparison to Theoretical Expectations

In Figure 17, the inferred masses and radii for uniform temperature models are compared with mass-radius curves for a wide variety of equations of state. The EOSs in this figure are labelled following the convention of Lattimer & Prakash (2001), and include cases with baryonic compositions (MS0, MS1, PAL6, GM3, AP4) together with one with a kaon-condensed core (GS1), all indicated by solid curves. In addition, two cases of self-bound stars composed of pure quark matter (SQM1, SQM3) are shown by dashed curves. The diagonal dashed line labelled "causality" is the approximate boundary $R > 3.04GM/c^2$ imposed by the requirement that the EOS never violates causality, and lines of constant R_{∞} are indicated by the dotted curves.

The cases MS0 and MS1 are representative examples of nucleonic field-theoretic EOSs, and AP4 is a state-of-the-art non-relativistic potential model EOS. The cases GM3, GS1 and PAL6 were chosen because of the softening they display above nuclear density: this is caused by hyperons and a kaon condensate for the field-theoretical EOSs GM3 and GS1, respectively, and by an extremely small incompressibility parameter for the schematic potential EOS

PAL6. Note that all these cases give values of $R \gtrsim 10$ km for 1.4–1.5 M_{\odot} , except in the case of extreme softening induced by a phase transition (viz. GS1). It is not inconceivable that even more compact stars could be obtained, but simultaneously satisfying the 1.44 M_{\odot} mass constraint (of PSR 1913+16) becomes very difficult.

The EOSs SQM1 and SQM3 illustrate the different behavior, relative to normal neutron stars, of self-bound configurations. Self-bound quark stars are subject to two constraints: first, that strange quark matter is the true ground state of matter (the so-called Witten's conjecture; Witten 1984), and second, that the maximum mass be larger than that of the most massive, accurately measured, neutron star (PSR 1913+16). The case SQM1 represents the most compact self-bound quark EOS that obeys these constraints. More compact configurations can be obtained, but only by violating these constraints and thus cannot be considered realistic in that connections to identifiable physics have not been established (Prakash, Baron & Prakash 1990; Lattimer et al. 1990).

The crosses in Figure 17, one for each of three assumed distances 51 pc, 61 pc and 71 pc, are the centroids of the allowed (hashed) regions whose extents are determined by the uncertainties we indicated in the four constraints (4)–(7). These regions imply configurations that are too small, for their masses, to be explained by a reasonable EOS, including those for self-bound strange quark matter. In addition, the masses are too small to fit theoretical expectations of neutron star masses from evolutionary considerations. The uncertainty in the distance is too small to affect these conclusions. These estimates of M and R are not sensitive to the precise way in which an optimum fit to the three sets of data (ROSAT, EUVE, and HST) is determined, i.e., on how much statistical weight should be assigned to each set of data.

We emphasize that a similar fit to all data is not possible for non-magnetized atmospheres composed by light elements (H, He). The four magnitude overestimate of the optical flux for an H or He atmosphere is too large to be reconciled without unacceptable deviations from the X-ray data. Pavlov et al. (1996) found that a magnetic field of 10^{12} G results in a decrease of 1.5 magnitudes (V band), compared to the non-magnetic case, with $R_{\infty}/D \approx 1.1$ km pc⁻¹. This correction is still much too small (by a factor of 20) to reconcile with the observed optical flux. More recent models with higher magnetic fields, appropriate for magnetars, seem to require hard power-law tails (Ho & Lai 2001; Özel 2001), and even a deficit of low energy photons, relative to the best blackbody fit, neither of which are observed in this case (although their models are for hotter sources). Zane, Turolla & Treves (2000) model accreting magnetized atmospheres and can reproduce the characteristic optical excess and lack of a hard tail, but at the expense of a luminosity two orders of magnitude larger, and temperature one order of magnitude, larger than we observe. It seems unlikely

that a magnetized hydrogen atmosphere can be reconciled with the observed spectral energy distribution.

However, X-ray spectra of neutron stars are often successfully fit with hydrogen or magnetic hydrogen atmospheres. In some cases hydrogen atmospheres are expected because the neutron star is accreting in a binary system (e.g., Rutledge et al. 2001). In other cases it may be that the X-ray spectrum alone provides insufficient leverage to distinguish between competing models (as is the case here).

6. NON-UNIFORM TEMPERATURE MODELS

Given the small value of the radius obtained from models in which the entire surface is assumed to be at the same temperature, we assess here the qualitative changes expected when non-homogeneous temperature distributions are considered. Our model atmospheres were built under the assumption of isotropic surface emission and low magnetic fields. The presence of a large magnetic field might cause significant anisotropy in the energy transfer. It has been shown that even without a magnetic field, the emerging spectrum will vary with the viewing angle with respect to the atmosphere normal. For emission from a small region of the star, these limb darkening effect causes substantial variation (see e.g. Zavlin et al., 1996). Thus the validity of our models is constrained to uniform or near-uniform temperature distributions. The careful modelling in the case of temperature anisotropies is beyond the scope of this paper. With this caveat in mind, we infer some qualitative results and indicate the expected trends by exploring simplified two-temperature models.

We consider here a simple two-component model in which a hot polar cap accounts for the X-ray emission and a cooler equatorial region explains the optical data. We denote the hotter temperature component with the subscript H and the cooler component with the subscript C, respectively. Then the fractional area covered by the hot component is $\alpha = (R_{H\infty}/R_{\infty})^2$ and $R_{\infty}^2 = R_{H\infty}^2 + R_{C\infty}^2$.

6.1. Two-Component Blackbody Models

A reduction in the fractional surface area of the hot region results in an increase in the optical flux. As an example, we consider a blackbody model with the component temperatures arbitrarily set to $T_{H\infty} = 55.3$ eV, the best-fit temperature for the uniform temperature blackbody model, and $T_{C\infty} = 20$ eV. In Figure 18 we show the spectral energy distribution for the one component model ($\alpha = 1$; dashed curve) and for a model with $\alpha = 0.2$ (solid

curve), which fully accounts for the optical emission. This behavior is easy to understand. The X-ray flux from the hot component dominates that of the cool one. But the optical flux from the hot component alone underestimates the measured optical flux by a factor f, which in the present case is f = 2.3. One then has

$$f - 1 = \frac{T_{C\infty}}{T_{H\infty}} \left(\frac{R_{C\infty}}{R_{H\infty}}\right)^2 . \tag{8}$$

The definition of α leads to

$$\alpha = \left[1 + (f - 1)\frac{T_{H\infty}}{T_{C\infty}}\right]^{-1} \simeq (1 + 3.6)^{-1} \simeq 0.2,$$
 (9)

for the example described above. The fractional contribution of the cool component to the total flux is

$$\left(\frac{T_{C\infty}}{T_{H\infty}}\right)^4 \left(\frac{R_{C\infty}}{R_{H\infty}}\right)^2 = (f-1)\left(\frac{T_{C\infty}}{T_{H\infty}}\right)^3 \simeq 0.06.$$
(10)

Since the cooler component peaks at longer wavelength, this is a strong upper limit to the fractional flux in the X-ray band (In actuality, less than 0.2% of the X-ray flux in this model arises from the cool component. In a model where $T_{C\infty} = T_{H\infty}/2$ the cool component contributes 2% of the soft X-ray flux.). This justifies using the same temperature for the hot component as that of the uniform temperature model. A small value of α results in a large increase in the predicted emitting surface area. In the present case the value of R_{∞}/D , which is proportional to $1/\sqrt{\alpha}$, has more than doubled.

Figure 19 shows the behavior of the inferred value of R_{∞} as the temperature of the cool component is lowered even more. The values of $T_{H\infty}$ and n_H are kept fixed and equal to the best fit to the uniform temperature BB model, which ensures reasonable agreement with EUVE and X-ray data. The inferred value of R_{∞} can be enlarged to as much as 15 km if $T_{C\infty}$ is reduced to 5 eV. The average visible surface area of the hot component in that case is only 6.5 % of the total surface of the star. The results essentially follow the simple relation given by equation (9), which shows that as $T_{C\infty} \to 0$, so does α .

In principle, it seems that one could make the star arbitrarily large by a dramatic decrease in the temperature of the cool component. However, the UV/optical spectrum arises in large part from the cooler region, and this allows us to place a firm lower limit on its temperature. For a sufficiently cool temperature, the spectrum will deviate from the Rayleigh-Jeans tail at the short wavelengths in the near UV part of the spectrum. This is shown in Figure 20, where we plot the far UV spectrum from the *HST*.

For a baseline, we subtract the best-fit, uniform-temperature BB model for the X-ray spectrum (Table 5), and then fit the residual flux with a series of BB spectra, normalized

to the total flux between 1310Å and 1650Å. The continuum slope of the far-UV spectrum cannot be reconciled with a BB with $T_{\infty} < 6.5$ eV at 90% confidence. If we further require that the BB curve pass through both the UV and optical photometric points (Figure 20 inset), then we can place a 1 σ confidence lower limit of 15 eV on the temperature of any cool surface that dominates the optical flux (this is the dotted vertical line in Figure 21). The value of R_{∞} for a two-component BB model could thus be increased relative to the uniform temperature BB model by, at most, the factor $1/\sqrt{\alpha}$, which from equation (9) for $T_{H\infty} = 55.3$ eV and $T_{C\infty} = 15$ eV, is about 2.4. This corresponds to an upper limit to R_{∞} of about 10 km.

6.2. Two-Component Atmospheric Models

In $\S 5.2.2$, we showed that uniform temperature models could fit the data, with formal best fit values for z of 0.34 (Si-ash) and 0.37 (Fe). Higher gravitational redshifts require a hotter and smaller surface, which underpredicts the optical flux. This flux could be supplied by additional cool components which do not emit at X-ray wavelengths. Thus two-component heavy element atmospheres are readily accommodated for higher redshift surfaces.

We follow the formalism developed in the previous section, but introduce the redshift dependence of $T_{H\infty}$. Figure 21 shows the permissable values of α . To the left of the dotted vertical line the models overpredict the optical flux. The region below the dash-dot line is excluded by the slope of the optical flux (§6.1). The dashed line marks where $T_{C,\infty}$ is half $T_{H,\infty}$; we exclude the region above this line since the shape of the X-ray spectrum will be affected by such a hot component. The allowable region lies between these bounds; in this region α increases with increasing z.

Using the parameterization of R_{∞}/D with z for these models, one can use these values for α to estimate the maximum R_{∞}/D to be about 0.21 km pc⁻¹ for both models (Figure 22). These represent only 60% increases in the radius over the best single component radius. The maximum radius occurs at high z, about 0.6; the increase in radius with z occurs because of the interplay between the increase in α and the decrease in R_{∞}/D with increasing z. We conclude that one cannot increase the radius arbitrarily, and that there is a clear upper limit on R_{∞}/D of about 0.21 km pc⁻¹ for these heavy element atmospheres. It is possible to approach the 10 km radius of the canonical neutron star using non-magnetic heavy element atmospheres. Given R_{∞}/D as a function of z, the minimum mass for the neutron star occurs near the minimum z, and the mass increases as z is increased, exceeding the canonical value of 1.4 M_{\odot} for $z \gtrsim 0.5$ (Figure 23).

It is also clear that the light-element atmospheres cannot be made consistent with the optical data by means of a non-uniform temperature model. The primary reason is that the best fits to X-ray data for these models overestimate the optical fluxes by a much greater factor than do heavy-element models. The lack of a dependence of $T_{H,\infty}$ on z (§5.2.2) means that one cannot remove the optical excess by increasing z, and so there is never any optical deficit to be made up with emission from a cool component.

The four analytic constraints for a simple two-component model are:

$$T_{H\infty}(R_{H\infty}/D)^2 + T_{C\infty}(R_{C\infty}/D)^2 = \begin{cases} 0.80^{+0.09}_{-0.05} \text{ eV} & \text{Fe} \\ 0.74 \pm 0.04 \text{ eV} & \text{Si - ash} \end{cases}$$
 (11)

$$n_{H,20} = (4.9 \pm 0.3)(1+z)^{-1} - (1.9 \pm 0.2)$$
(12)

$$T_{H\infty} = \left\{ \begin{array}{ll} (157 \pm 2) - (96 \pm 3)n_{H,20} + (17 \pm 1)n_{H,20}^2 \text{ eV} & \text{Fe} \\ (162 \pm 4) - (91 \pm 4)n_{H,20} + (15 \pm 1)n_{H,20}^2 \text{ eV} & \text{Si - ash} \end{array} \right\} (13)$$

$$T_{H\infty} = \begin{cases} (157 \pm 2) - (96 \pm 3)n_{H,20} + (17 \pm 1)n_{H,20}^{2} \text{ eV} & \text{Fe} \\ (162 \pm 4) - (91 \pm 4)n_{H,20} + (15 \pm 1)n_{H,20}^{2} \text{ eV} & \text{Si - ash} \end{cases}$$
(13)
$$T_{H\infty}^{4} (R_{H\infty}/D)^{2} + T_{C\infty}^{4} (R_{C\infty}/D)^{2} = \begin{cases} (6147 \pm 839) \exp[(1.46 \pm 0.07) n_{H,20}] (\text{eV})^{4} & \text{Fe} \\ (3719 \pm 842) \exp[(1.58 \pm 0.10) n_{H,20}] (\text{eV})^{4} & \text{Si - ash} \end{cases}$$
(14)

In addition, we have the constraints

$$R_{C\infty}^2 > 0 \tag{15}$$

$$T_{H\infty}/2 > T_{C\infty} > 15 \text{ eV} \,.$$
 (16)

Note that constraint (15) is equivalent to setting a lower limit to z. Additionally, we limit possible values of M and R by causality. As previously, radii are in km and distances in pc.

The allowable region in the mass-radius diagram from the two-temperature model defined by equations 11–16 is shown in Figure 24. The shaded region indicates the allowed values of M and R at 90% confidence, for an assumed distance of 61 pc, including the uncertainty in the distance. The results for M and R scale with D.

As expected, the addition of the second component enlarges the allowed regions for Mand R, but there exists a lower limit on the compactness. We note that the largest radii occur for the most extreme temperature variations and for the largest allowable distance. While this simplified model shows the qualitative trends, only a more detailed, and self-consistent, analysis of surface inhomogeneities can establish realistic limits.

DISCUSSION AND OUTLOOK

We have calculated a series of neutron star model atmospheres for different chemical compositions, which have been used as input models to fit multiwavelength spectrophotometric observations of the nearby compact object RX J185635-3754. We have investigated the constraints that exist among relevant parameters which most influence the predicted spectra. The *ROSAT* X-ray data alone are insufficient to adequately constrain these parameters, but the combination of X-ray and UV/optical observations does allow significant constraints to be imposed. Our main conclusions are:

- A uniform temperature blackbody is excluded by the multiwavelength data.
- Non-magnetized light-element atmospheres are excluded since they are incompatible with combined optical and X-ray observations. Results for magnetic H-atmosphere models (Pavlov et al. 1996) indicate that fields of order 10¹² G or less are unlikely to change this result.
- The simplest uniform temperature heavy-element atmospheric models indicate that $T_{\infty} \approx 45 \text{ eV}$, $R_{\infty} \approx 8 \text{ km}$, $M \approx 0.9 \text{ M}_{\odot}$, and $R \approx 6 \text{ km}$. This mass and radius are too small to be consistent with any equation of state in common use, including even that of self-bound strange quark matter. This value for R_{∞} represents a lower limit in models with temperature inhomogeneities.
- For heavy-element compositions or a blackbody, the accumulated optical data yields- $T_{\infty}(R_{\infty}/D)^2 = 0.7 \pm 0.1$ eV because it falls on the Rayleigh-Jeans tail of the emission.
- That the optically flux does not significant deviate from a Rayleigh-Jeans behavior implies that the temperature of the star is greater than 15 eV. Combined with upper limit to the distance, this indicates that $R_{\infty} \leq 16.5 \text{ km}$ ($M \leq 2.15 \text{ M}_{\odot}$) at 1 σ .
- A simple two-component blackbody model provides an acceptable fit of the data for $R_{\infty} \leq 10 \text{ km } (M \leq 1.3 \text{ M}_{\odot}).$
- Simple two-component heavy-element atmospheres can also provide acceptable fits, provided z exceeds 0.34 (Si-ash) or 0.37 (Fe). In these cases, neutron star configurations up to $R \approx 10$ km can be allowed. Further investigation of models including inhomogeneous thermal emission are thus worth pursuing.
- The uncertainty in the luminosity is less than those of any other neutron star for which thermal emission is believed to be seen. We find, for acceptable models, $L = 1.5 \pm 0.2 \cdot 10^{31}$ erg/s including the uncertainty in the distance. As shown in Figure 25, which summarizes the existing data concerning neutron star cooling, the location of RX J185635-3754 is consistent with both the spin-down luminosities and ages of the other objects shown.

7.1. Causes and Implications of Non-Uniform Temperature Distributions

If the surface is a blackbody, or if radius is to approach the predictions from realistic equations of state, then it appears that the surface temperature must be non-uniform. Here we briefly discuss some possibilities for generating these anisotropies, and whether we should expect to detect them directly.

The magnetic field itself could generate a large temperature gradient (Shibanov & Yakovlev 1996) if its strength exceeded 10^{12} G. Note, however, that under this assumption, the hot polar region contains about 80%-90% of the surface area, while the analysis presented in §6 implies the opposite. Nevertheless, this scenario might account for about a 20% increase in the radius for a model wherein the contribution of the hot component only slightly underestimates the optical fluxes. This still leaves the estimated masses and radii well below theoretical expectations.

Another possible source of temperature anisotropy is rotation. Miralles et al. (1993) investigated rotationally produced temperature anisotropies, while the effect of rotation on the cooling of neutron stars has been investigated by Schaab & Weigel (1998). A neutron star with an isothermal core, rotating with nearly its Keplerian frequency, might have a polar temperature up to 30% higher than the equatorial temperature, due to the angular dependence of the surface gravity. This effect also seems smaller than what is required.

A relevant question is whether or not a neutron star with an inhomogeneous surface temperature, spinning with typical periods and with a magnetic field of the order of 10^{12} G, should be seen as an X-ray pulsar. As shown in §3, no modulation has been detected at a level above 6%. This is not, however, a strong argument against a large surface temperature anisotropy since a star as compact as we have inferred $(GM/Rc^2=0.22-0.32)$ has a maximum allowed pulse fraction of less than 10% (Page 1995; Page & Sarmiento 1996; Psaltis, Özel & DeDeo 2000). This limit is even lower if the magnetic and rotation angles are not perpendicular or if the hot spot occupies a large fraction of the surface. The lack of modulation in the signal is due to general relativistic deflections which expose a large fraction of the surface to an observer at infinity. Any modulation detected in the future by more sensitive observations would thus be extremely useful for setting upper limits to the compactness of the object, as has been claimed for other isolated neutron stars (Wang et al. 1999). A related effect (Psaltis, Ozel & DeDeo 2000) is that, for z > 0.25, the spectroscopically inferred polar-cap surface area is, at most, 10% different than its intrinsic area. Therefore our estimates of the radius from the inferred surfaces are barely affected by the observer's inclination or the opening angle of the polar cap.

7.2. Can RX J185635-3754 Be Accreting?

Accretion can be a source of heating, and is a mechanism which can generate relatively large surface temperature variations. Earlier, we dismissed Bondi-Hoyle accretion onto a non-magnetized star as an unlikely source of the observed luminosity because of the large space velocity (unless the local interstellar density is about 10⁴ cm⁻³). However, as we also pointed out, it does not take much accretion to make a hydrogen atmosphere.

An independent check of the accretion hypothesis is provided by the ASCA data. Nelson et al. (1995) argue that a magnetized neutron star accreting from the interstellar medium could emit 0.5 - 5% of its accretion luminosity in the form of a cyclotron line between 5 and 20 keV. The ASCA spectra, however, place an upper limit to the flux between 1.5 and 12 keV of 0.5% of the total X-ray flux. If the soft X-ray flux were entirely due to accretion, we could exclude magnetic field strengths between about 1 and 7×10^{12} G. Conversely, if the star is magnetized in this range, then no more than about 10% of the soft X-ray luminosity could be due to accretion.

If RX J185635-3754 is a typical 10⁶ year old neutron star, then it is most likely magnetized. At the 10⁶ year age suggested by its parallax and proper motion (Walter 2001), it is likely to be in the ejector phase (Treves et al. 2000). If older, then it is most likely in the propeller phase like most neutron stars (Colpi et al. 1998). In either case, the large magnitude of the magnetic fields implied by the ejector and propeller phases make it extremely unlikely that it could accrete from the interstellar medium, and may explain why the atmosphere is not dominated by hydrogen.

Another indication that this star is not significantly accreting, but that it might be magnetized, is based on the deep VLT image released by van Kerkwijk & Kulkarni (2000). This image shows a classic bow-shock nebula. The presence of a bow-shock suggests that this is a magnetized neutron star with a relativistic wind, as observed in the pulsars PSR 1957+20 (Kulkarni & Hester 1988) and PSR 2224+65 (Cordes, Romani, & Lundgren 1993). Analysis of the broadband F606W images of this region (Walter & Wijers 2001) show that the standoff distance between the target and the apex of the nebula is about 1 arcsec (60 AU at a distance of 60 pc). Pressure balance between the interstellar medium and the relativistic wind produces a standoff distance of $26 \sqrt{B_{12}/n}/(P^2v_{100})$ AU, where B_{12} is the magnetic field strength in units of 10^{12} G, n is the density of the interstellar medium in cm⁻³, P is the rotation period of the neutron star in seconds, and v_{100} is the velocity of the neutron star relative to the interstellar medium in hundreds of km s⁻¹. While other mechanisms can cause a bow-shock geometry, their spatial scales are not correct. In the absence of a relativistic wind, the ram pressure of the interstellar medium on the neutron star's magnetic field leads to a standoff distance of $0.02(B_{12}/(\sqrt{n}v_{100}))^{1/3}$ AU. An ionization front has an

expected radius of $4000\sqrt{L_{31}/(nv_{100})}$ AU, where L_{31} is the luminosity of ionizing photons in units of 10^{31} erg s⁻¹, and there should be a partially ionized zone extending out about twice as far.

Walter & Wijers (2001) estimate from the VLT image that RX J185635-3754 likely has $B_{12} > 0.4$ G, and P > 0.5 s. These values are fairly typical for old pulsars. In this case, RX J185635-3754 might be a dead or misaligned radio-pulsar. If so, RX J185635-3754 has likely never accreted after the initial fallback from the supernova, which could explain why the surface is apparently dominated by heavy elements and not hydrogen. A typical pulsar magnetic field may also not be large enough to substantially affect the emergent spectra of heavy-element atmospheres (Rajagopal, Romani & Miller 1997). In fact, (Rajagopal, Romani & Miller 1997) show that the spectral energy distributions of magnetized ($10^{12} - 10^{13}$ G) Fe atmospheres are closer to blackbodies than are those of unmagnetized Fe atmospheres. In light of the small differences between our Fe atmospheres and the BB model fits, it is unlikely that the presence of magnetic fields of this magnitude will substantially affect our conclusions.

7.3. Epilogue

The simplest interpretation consistent with the data presented here is of uniform thermal emission from a compact object with a heavy element atmosphere. The object appears to be smaller than any canonical neutron star or self-bound strange star and less massive than the current supernova paradigm would allow. Allowing surface temperature inhomogeneities still results in a relatively compact object, but one which no longer excludes all models. A two-component blackbody is acceptable, as are one or two-temperature component heavy element atmospheric models. The most robust results from the atmospheric modelling appear to be a restriction of the star's redshift (z > 0.3) and a surface composition devoid of light elements. The large inferred compactness is not completely without precedent: for example, Wanajo et al. (2001) have recently argued that very compact stars ($z \sim 0.58$) are required to explain the yield of r-process elements, if produced in a neutrino-driven wind.

The small inferred radius aside, the star does not appear unusual. The bow-shock image (§7.2) suggests that this may be a common pulsar viewed from the side. The luminosity (Figure 25) appears normal for its age.

Data such as these open the exciting possibility that observations can offer meaningful constraints on the EOS. The uncertainties in the values we have obtained for the mass and radius of RX J185635-3754 are mostly limited by the relatively poor spectral resolution

of the X-ray data. Nevertheless, this object does afford a clear opportunity to measure these properties accurately when higher resolution X-ray spectra from the Chandra and XMM-Newton observatories become available. Most importantly, such spectra may reveal X-ray lines which would allow the unambiguous determination of the star's atmospheric composition and redshift. High S/N far-UV spectra in the 1500-2000Å range can also be used to search for a H I Ly α line, which may be present even if the overall composition is dominated by heavy elements. And if the spectra reveal no lines, perhaps we should recall that the spectrum of a self-bound strange quark matter star is likely to be a pure thermal spectrum. RX J185635-3754 has yet to reveal all its secrets.

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- Fig. 1.— The net spectrum of RX J185635-3754 observed with the *EUVE* SW detector. For comparison we show the blackbody (BB) fits to *ROSAT* data alone (Table 5; dashed curve) and to the combined optical and X-ray data (Table 6; solid curve).
- Fig. 2.— The background-subtracted count spectrum of RX J185635-3754 observed with ASCA. The lower panel shows the combined SIS spectrum; the combined GIS spectrum is in the upper panel.
- Fig. 3.— Emergent (non-redshifted) spectral flux distributions for selected compositions for $g_{14} = 2.43$. Curves are labelled by their $\log T_{eff}$ and composition (BB dotted lines, Fe solid lines, He dashed lines, and H dot-dashed lines). The temperatures chosen for the spectra in the left panel correspond to those of Figure 2 in Rajagopal & Romani (1996), and those in the right panel correspond to those in Figure 5 of Zavlin, Pavlov & Shibanov (1996).
- Fig. 4.— Emergent (non-redshifted) spectral flux distributions from Fe atmospheres with $\log T_{eff} = 5.6$ and the indicated surface gravities g_{14} . The BB flux is also shown for comparison.
- Fig. 5.— The best-fit Fe model (upper panel) and residuals (lower panel) to the *ROSAT* PSPC data. The parameters of the fit are listed in Table 5.
- Fig. 6.— The effects of varying the redshift on the best fits to the ROSAT PSPC data. Thick (thin) lines indicate T_{∞} (n_H) for both Fe (dashed lines) and Si-ash (solid lines) models. Errors (plotted) are about ± 1.8 eV for T_{∞} and ± 0.2 for $n_{H,20}$, at a given z. For clarity, the errors on $n_{H,20}$ are indicated only on the Si-ash points.
- Fig. 7.— The correlations between n_H and T_{∞} for the best-fit Fe (dashed line) and Si-ash (solid line) models to the ROSAT PSPC data at fixed values of the redshift. The redshift decreases from left to right. For comparison, the best-fit BB model is indicated by a diamond.
- Fig. 8.— The spectral flux distribution in the 1 eV 1 keV range, for blackbody models. The dashed curve is the best-fit model to the ROSAT PSPC data, while the solid curve is the best-fit model to the combined optical, EUVE and ROSAT data. Both models are generated for z=0.305. The ROSAT PSPC data are not displayed because the energy redistribution of the response matrix does not permit a model-independent flux to be determined. The EUVE data are denoted by the error bars in the 0.12–0.18 keV range. The mean STIS flux is shown; the spectrum is shown on a larger scale in Figure 20. Error bars on all points denote the photometric uncertainties; horizontal bars denote the band widths for the broadband

photometry.

- Fig. 9.— Same as Figure 8 but for a pure Fe atmosphere. The dotted curve is the best multiwavelength fit at the optimal z.
- Fig. 10.— Same as Figure 9 but for a Si-ash atmosphere. The dotted curve (barely distinguishable from the solid curve) is the best multiwavelength fit at the optimal z.
- Fig. 11.— Same as Figure 8 but for a pure H atmosphere. Note the difference in scale. In this case (and that for He, not shown), no acceptable joint fits of optical, EUVE and ROSAT data are possible. The absorption feature in the models near 0.08 keV is H I Lyman α for z=0.305.
- Fig. 12.— The acceptable regions in the $R_{\infty}/D kT_{\infty}$ plane for the blackbody fits. Three σ confidence contours are drawn. The thick contour denotes the best fit to the multiwavelength data, with the four data sets given equal weight. The best fits to the individual data sets are also shown. Contours of constant n_H (=1, 2, and 3×10^{20} cm⁻² from bottom to top) are overplotted. Note that the regions allowed by the X-ray data follow the n_H contours, while the optical and UV data, taken longward of the peak of the emission, allow regions of approximately constant R_{∞}/D . The thick dots mark the limits of the region explored. Note that at 3σ confidence the regions allowed by the PSPC and the optical/UV data do not intersect.
- Fig. 13.— Same as Figure 12 but for a pure Fe atmosphere. At 3σ confidence the regions allowed by the PSPC and the optical/UV data do not intersect, but the discrepancy is less than for the blackbody model.
- Fig. 14.— Same as Figure 12 but for a Si-ash atmosphere. At 3σ confidence the regions allowed by the PSPC and the optical/UV data do intersect.
- Fig. 15.— Same as Figure 12 but for a pure H atmosphere. The X-ray and optical/UV regions cannot be reconciled. For clarity, we plot 4σ confidence contours. The best formal fit region lies near the optical 4σ confidence contour, at $kT_{\infty}=26$ eV.
- Fig. 16.— The effects of redshift on the derived value of R_{∞}/D , for the Fe (right panel) and Si-ash (left panel) models. The solid curves are obtained by fitting the ROSAT PSPC data with a uniform-temperature model, and the filled bands are determined from the optical observational constraint (7). Their intersections denote the combined acceptable values of z and R_{∞}/D . In each case, the dashed lines bracketing the solid lines, and the width of the shaded bands, indicate the uncertainties in R_{∞}/D .

Fig. 17.— Mass-radius diagrams for the uniform-temperature heavy element atmosphere models. Upper and lower panels are for Fe and Si-ash compositions, respectively. Solid and dashed curves are for equations of state labelled following Lattimer & Prakash (2001). The dashed line labelled "causality" is the compactness limit set by requiring equations of state to be causal. Dotted lines are contours of fixed R_{∞} . The crosses denote masses and radii of models which best fit the optical and X-ray data, for the indicated distances, and the hatched regions surrounding them include the nominal errors indicated in the constraint relations 4–7.

Fig. 18.— The spectral flux distribution for a two-component BB model with $T_{H\infty} = 55.3 \text{ eV}$ and $T_{C\infty} = 20 \text{ eV}$. The dashed line indicates the best-fit uniform-temperature BB model ($\alpha = 1$), and the solid curve is the best 2-component component fit, with $\alpha = 0.22$. The cool component makes no significant contribution to the X-ray flux. Other notation is the same as in Figures 8-11.

Fig. 19.— The effective increase in R_{∞} that can be obtained by progressively lowering $T_{C\infty}$ in the two-component BB model fit to the combined optical and X-ray data. The thick solid curve is for the assumed 61 pc distance, and dashed curves indicate $^{+9}_{-8}$ pc deviations from this. The thin solid curve shows α as a function of $T_{C\infty}$. $T_{H\infty}$ is fixed to 55.3 eV, the value for the best-fit uniform-temperature BB fit. The vertical dotted line indicates the lower limit to $T_{C\infty}$, 15 eV, from the slope of the optical and UV fluxes. This formulation assumes that the cool component does not contribute any significant flux at X-ray wavelengths, and so breaks down as $T_{C\infty}$ approaches $T_{H\infty}$.

Fig. 20.— The far UV spectrum from STIS with $\pm 1\sigma$ envelope contours (dotted lines). The data have been smoothed with a Fourier filter and a 7-pixel (4.1Å) running mean. Uncertainties are particularly large in the vicinity of the subtracted geocoronal H I Lyman- α and O I emission at 1216 and 1300 Å, respectively. There are no statistically significant emission or absorption features in this spectrum. The lower solid curve is the baseline BB fit ($T_{\infty} = 55.3 \text{ eV}$, $R_{\infty}/D=0.070 \text{ km pc}^{-1}$). The upper three curves are BB fits to the residual flux above the baseline for $T_{\infty}=5$, 10, and 25 eV, respectively. The inset shows the optical-UV spectral energy distribution on a log-log plot with flux units identical to that of the main plot. The crosses show the mean STIS flux and the mean F170W, F300W, F450W, and F606W fluxes, together with the relevant wavelength ranges. The smooth curves are the extrapolations of the $T_{\infty}=5$, 10, and 25 eV BB curves to longer wavelengths.

Fig. 21.— Contours of α as a function of the gravitational redshift z and the temperature of the cool component $T_{C,\infty}$. The vertical dotted line represents the value of z below which there is an optical excess. The horizontal dashed lined, at $T_{C,\infty}=22$ eV, is the coolest permissable

temperature based on the optical spectrum. The diagonal dashed line is at half of $T_{H,\infty}$, and represents approximately the point above which the cool component begins to noticeably affect the shape of the high energy spectrum. Allowable regions lie within the wedge between the dashed lines. Kinks in the curves are a consequence of the discrete grid used.

- Fig. 22.— R_{∞}/D as a function of z for the two-component heavy element atmospheres. The vertical dotted lines represent the lower limits for z; below this the model predicts too much optical emission. Addition of a cool component can only increase R_{∞}/D by up to about 60% over the single temperature component value, and then only for very high values of z.
- Fig. 23.— Mass as a function of z for the two-component heavy element atmospheres. Masses are plotted only for points for redshifts for which the two-component model is valid.
- Fig. 24.— Similar to figure 17, but for two-temperature fits to X-ray and optical data. The shaded region is the 90% confidence region allowed by the constraints (11)–(16), for an assumed distance of 61 pc.
- Fig. 25.— Cooling curve observations for neutron stars for which thermal emission is believed to be seen. Except for RX J185635-3754, data are from Pavlov (2001; private communication) and ages are the standard pulsar spin-down times, or the remnant age if known. In many cases, luminosities derived from different assumed compositions are shown with errors (mostly resulting from distance uncertainties). MH indicates magnetic hydrohen atmospheres; BB indicates blackbody fits. The luminosity for RX J185635-3754 is that of the heavy element atmospheres; the two-component blackbody model has a similar luminosity.

Table 1. ROSAT Observation Log

ROR	Instrument	Start (UT)					Stop Length (ksec)		Length (ksec)	
200497 400612 400864	PSPC HRI HRI	1992 1994 1997	7	Oct	10:06 13:06 14:34	8	Oct	5:33 22:22 9:39	6.3 17.8 29.1	

Table 2. $\it EUVE$ Observing Log

Start (1997	Stop UT)	Length (ksec)
18 June 04:26	24 Jun 23:29	195
26 June 09:59	28 Jun 22:07	60
30 June 22:24	3 Jul 12:42	77
9 July 00:33	16 Jul 00:23	180
16 July 21:44	19 Jul 08:50	90

Table 3. *HST* Observations

Root	Filter	exposure (sec)	Start (UT)		$\begin{array}{c} f_{\lambda} \\ (\mathrm{erg} \ \mathrm{cm}^2 \ \mathrm{s}^{-1} \ \mathrm{\mathring{A}}^{-1}) \end{array}$	σ
U3IM010x, x=1-4	F606W	4400	1996 Sep 30	2:25	1.29×10^{-19}	16.7
U51G010x, x=1-8	F606W	7200	$1999~\mathrm{Mar}~30$	0:35	1.51×10^{-19}	23.2
U51G020x, x=3-6	F606W	5190	$1999~{\rm Sep}~16$	7:51	1.42×10^{-19}	19.2
U625010x, x=1-6	F606W	7400	$2001~\mathrm{Mar}~24$	19:32	1.37×10^{-19}	22.0
U51G040x, x=1-8	F450W	7200	1999 May 26	17:20	4.26×10^{-19}	15.3
U3IM010x, x=5-6	F300W	2400	$1996 \ \mathrm{Oct} \ 12$	5:29	1.90×10^{-18}	7.3
U51G020x, x=1-2	F300W	2600	$1999~{\rm Sep}~16$	6:15	2.48×10^{-18}	6.9
U51G030x, x=1-4	F170W	10800	1999 May 24	16:43	1.45×10^{-17}	5.8

Table 4. Si-ash Composition^a

Element	Si	Ar	Ti	Mn	Ni	S	Ca	Cr	Fe
Abundance (%)	11	3	1	1	2	10	3	1	68

^afrom Arnett (1996)

Table 5. ROSAT PSPC Spectral Fits

Model	z	$\begin{array}{c} n_H \\ (10^{20} \text{ cm}^{-2}) \end{array}$	T_{∞} (eV)	χ_{ν}^{2a}	$\frac{R_{\infty}/D}{(\text{km pc}^{-1})}$	<i>D</i> ^b (рс)	R_{∞}^{c} (km)
		Fits with	z fixed at the	he non	ninal value of 0.3	305	
BB H He Fe Si-ash		1.73 ± 0.13 2.36 ± 0.10 2.40 ± 0.10 2.07 ± 0.14 1.88 ± 0.11	55.3 ± 5.5 13.2 ± 1.6 13.3 ± 1.6 33.7 ± 1.5 42.6 ± 1.6	1.43 1.44 1.44 1.86 1.72	0.070 ± 0.015 2.19 ± 0.57 2.38 ± 0.60 0.290 ± 0.047 0.147 ± 0.020	186 ± 40 6.0 ± 1.6 5.5 ± 1.4 45 ± 7 89 ± 12	4.3 ± 1.1 134 ± 35 145 ± 37 18 ± 3 9.0 ± 1.2
		E	Best fits with	z allov	wed to vary		
H Fe Si-ash	0.43 0.64 0.54	2.44 ± 0.10 0.96 ± 0.14 1.17 ± 0.16	12.7 ± 1.5 80.0 ± 3.5 69.1 ± 4.0	1.43 1.49 1.45	2.52 ± 0.62 0.028 ± 0.002 0.036 ± 0.005	5.2 ± 1.3 466 ± 33 363 ± 50	154 ± 38 1.7 ± 0.1 2.2 ± 0.3

^aReduced χ^2 for 86 degrees of freedom.

 $^{^{\}rm b}{\rm Distance}$ assuming $R_{\infty}{=}13.05$ km.

 $^{{}^{\}rm c}R_{\infty}$ assuming distance = 61^{+9}_{-8} pc.

Table 6. Parameters from Multiwavelength Fits^a

Model	n_H $(10^{20} \text{ cm}^{-2})$	T_{∞} (eV)	$\frac{R_{\infty}/D}{(\text{km pc}^{-1})}$	$T_{\infty}(R_{\infty}/D)^2$ (eV (km pc ⁻¹) ²)	Luminosity $(10^{31} \text{ erg s}^{-1})^{\text{c}}$	P_{OX}^{b}
BB H	$2.2^{+0.3}_{-0.4}$ 1.0 ± 0.1	48±2 26±1	0.11 ± 0.01 0.27 ± 0.01	$0.60^{+0.05}_{-0.4}$ 1.94 ± 0.01	$1.55^{+0.23}_{-0.17}$ 0.6 ± 0.01	3×10^{-4} $< 10^{-14}$
Fe	1.8 ± 0.2	44±1	0.13 ± 0.01	0.75 ± 0.05	$1.41^{+0.08}_{-0.06}$	7×10^{-7}
Si-ash	$1.9^{+0.3}_{-0.2}$	45^{+2}_{-1}	0.13 ± 0.01	$0.74^{+0.04}_{-0.05}$	$1.63^{+0.14}_{-0.21}$	0.53

 $^{^{\}mathrm{a}}3\sigma$ ranges, assuming $z{=}0.305$. Weighting of the data is discussed in the text.

 $^{^{\}mathrm{b}}\mathrm{The}$ likelihood that the X-ray and optical parameters are the same.

^cUncertainty does not include uncertainty in distance.

Table 7. Parameter Constraints

Parameter	Analytic Fit	Joint Acceptable Region	units
-			
	S	i-ash model	
n_H	1.84 ± 0.07	1.92 ± 0.05	10^{20} cm^{-2}
z	0.31 ± 0.12	$0.34_{-0.03}^{+0.01}$	
${ m T}$	45.2 ± 6.0	45 ± 1	eV
R_{∞}/D	0.128 ± 0.021	$0.132 {}^{+.011}_{008}$	$\mathrm{km}\ \mathrm{pc}^{-1}$
		Fe model	
$\overline{n_H}$	1.66 ± 0.11	1.81 ± 0.06	10^{20} cm^{-2}
z	0.38 ± 0.12	$0.37 ^{+0.04}_{-0.03}$	
${ m T}$	44.3 ± 3.8	44.0 ± 1	eV
R_{∞}/D	0.134 ± 0.021	$0.139 {}^{+.010}_{012}$	$\mathrm{km}\;\mathrm{pc}^{-1}$

















































